

Lesson 2: Business Applications of Rate of Change

We live in a world that demands that we search out the **best** - avoid the **worst** – earn the **maximum** – spend the **minimum**....

Through mathematical modeling, calculus can be used to establish optimal conditions for situations that have competing variables. This process is referred to as “**Optimization**”.



<http://christconquers.files.wordpress.com/2011/06/canadian-money.jpg>

Example #1:



<http://gowebnow.net/epress/wp-content/2011/02/money-increase.jpg>

Consider a company that manufactures widgets.

x = the number of widgets (in THOUSANDS)

$p(x)$ = the **price** of “ x ” widgets

$C(x)$ = the **Cost** to manufacture “ x ” widgets

$R(x)$ = the **Revenue** made from “ x ” widgets

$$= x \cdot p(x)$$

$P(x)$ = the **Profit** earned from “ x ” widgets

$$= R(x) - C(x)$$

If you are provided with the functions for **Revenue** and **Cost**, you can then determine a wide variety of values.

Given that $R(x) = 100x - 0.001x^2$, and $C(x) = 10x + 0.0001x^2$, where x is the number of widgets (in THOUSANDS),
(In economic terms, this limited timeframe would be considered “the short run”.)

- a) How much **Profit** is made when 6000 widgets are sold?
(Hint: Profit = Revenue – Cost)

$$\text{Profit} = P(x) = R(x) - C(x)$$

$$P(6) = 516$$

The profit is \$516 when 6000 widgets are sold.



- b) How many widgets must be produced for the company to reach the **Break Even Point?**
(i.e. the point where no Profit is made OR where Revenue = Cost)

For Break Even point, set Profit $P(x) = 0$.

$P(x) =$ → Solve using the quadratic formula

So, $x =$ (approx) 4.5666 or -15.7666 (a negative # is an inadmissible value)
AND x is measured in THOUSANDS, so x represents 4.5666 x 1000 widgets.

For the company to Break Even, they must produce approximately 4567 widgets.

c) Determine the Marginal Cost for producing the 6000th widget.

Marginal Cost or $C'(x)$ is the incremental cost for producing each additional widget.
This value can be easily calculated by determining the derivative of the Cost function.

, then
And

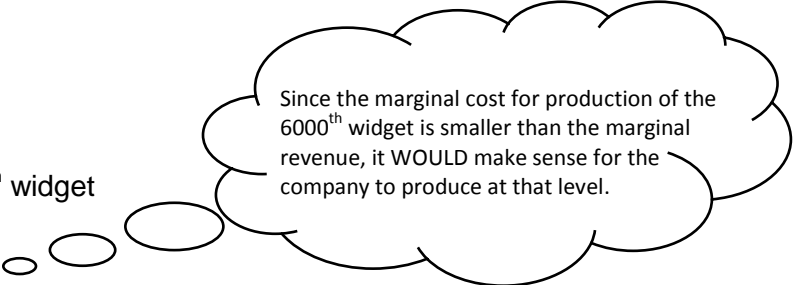
The Marginal Cost for the 6000th widget to be produced is \$926.

d) Determine the Marginal Revenue for the 6000th widget.

Marginal Revenue or $R'(x)$ is the incremental change in revenue for each additional widget sold. This value can be calculated by determining the derivative of the Revenue function.

, then
And

The Marginal Revenue for the 6000th widget to be sold is \$958.



Since the marginal cost for production of the 6000th widget is smaller than the marginal revenue, it WOULD make sense for the company to produce at that level.

e) When will the **Profit** be maximized? What will the company's profit be at this point?

For MAXIMUM value, find derivative of $P(x)$, then set $P'(x) = 0$.

Profit = , then

$$5.6 = x$$

The Profit will be maximized when 5600 widgets are sold.

$$\begin{aligned} &= -156.8 + 313.6 + 360 \\ &= 516.8 \end{aligned}$$

The maximum profit will be \$516.80.



Provide complete solutions to **Lesson 2 questions**: (complete solutions found in Appendix A)
(answer key found in Appendix B)



1. A retailer normally sells 1200 pairs of running shoes for \$60 a pair. A survey shows that sales will increase by 200 for each \$5 decrease in price. Based on this pattern, how many pairs of shoes should be sold to maximize the revenue?
2. A toy store sells 2000 Lego sets per day for \$25 each. A survey shows that the daily sales will increase by 140 for each \$2 decrease in price. Based on this pattern, how should Lego sets be priced to maximize the revenue?
3. A wood screw manufacturer has determined that the total cost $C(x)$ of operating a factory is $C(x) = 0.5x^2 + 45x + 10000$, where x is the number of wood screws produced in **thousands**. Determine the production level that minimizes the average cost.
(Note: Average Cost is found by dividing Total Cost by # of items... or —)
4. A soft drink company estimates that the cost, $C(x)$, in dollars of producing x litres of soda per day is $C(x) = 0.0005x^2 + 4x + 4000$. Find the marginal cost when 5000L of soda is produced.
5. A local manufacturer sells cedar deck chairs. The company can sell x chairs at a price of $p(x) = 100 - x$ dollars, where the cost function is, $C(x) = 30 + 19x^2$. Determine the number of chairs that need to be produced to break-even.