# Lesson 2: Business Applications of Rate of Change


http://christconquers.files.wordpress.com/2011/06/canadian-money.jpg

We live in a world that demands that we search out the **best** - avoid the **worst** – earn the **maximum** – spend the **minimum**….

Through mathematical modeling, calculus can be used to establish optimal conditions for situations that have competing variables. This process is referred to as **"Optimization".**

Example #1:

http://gowebnow.net/epress/wp-content/2011/02/money-increase.jpg

Consider a company that manufacturers widgets.

*x* = the number of widgets (in THOUSANDS)
*p(x)* = the **price** of "x" widgets
*C(x)* = the **Cost** to manufacture "x" widgets
*R(x)* = the **Revenue** made from "x" widgets
$\quad = x \cdot p(x)$
*P(x)* = the **Profit** earned from "x" widgets
$\quad = R(x) - C(x)$

If you are provided with the functions for **Revenue** and **Cost**, you can then determine a wide variety of values.

Given that R , and , where .
(In economic terms, this limited timeframe would be considered "the short run".)

a) How much **Profit** is made when 6000 widgets are sold?
(Hint: Profit = Revenue – Cost)

Profit = $P(x)$ $= R(x) - C(x)$

—

$P(6)$

= 516

The profit is $516 when 6000 widgets are sold.

b) How many widgets must be produced for the company to reach the **Break Even Point?**
(i.e. the point where no Profit is made OR where Revenue = Cost)

For Break Even point, set Profit $P(x) = 0.$

P(x) = → Solve using the quadratic formula

So, x = (approx) 4.5666 or -15.7666 (a negative # is an inadmissible value)
AND x is measured in THOUSANDS, so x represents 4.5666 x 1000 widgets.

For the company to Break Even, they must produce approximately 4567 widgets.

c) Determine the Marginal Cost for producing the 6000$^{th}$ widget.

***Marginal Cost*** *or C'(x) is the incremental cost for producing each additional widget. This value can be easily calculated by determining the derivative of the Cost function.*

, then
And

The Marginal Cost for the 6000$^{th}$ widget to be produced is $926.

d) Determine the Marginal Revenue for the 6000$^{th}$ widget.

***Marginal Revenue*** *or R'(x) is the incremental change in revenue for each additional widget sold. This value can be calculated by determining the derivative of the Revenue function.*

, then
And

The Marginal Revenue for the 6000$^{th}$ widget to be sold is $958.

Since the marginal cost for production of the 6000$^{th}$ widget is smaller than the marginal revenue, it WOULD make sense for the company to produce at that level.

e) When will the **Profit** be maximized? What will the company's profit be at this point?

For MAXIMUM value, find derivative of *P(x),* then set *P'(x)* = 0.

Profit = , then

5.6 = *x*

The Profit will be maximized when 5600 widgets are sold.

= *-156.8 + 313.6 + 360*
= *516.8*

The maximum profit will be $516.80.

Provide complete solutions to **Lesson 2 questions**: *(complete solutions found in Appendix A)*
*(answer key found in Appendix B)*

1. A retailer normally sells 1200 pairs of running shoes for $60 a pair. A survey shows that sales will increase by 200 for each $5 decrease in price. Based on this pattern, how many pairs of shoes should be sold to <u>maximize the revenue</u>?

2. A toy store sells 2000 Lego sets per day for $25 each. A survey shows that the daily sales will increase by 140 for each $2 decrease in price. Based on this pattern, how should Lego sets be priced to <u>maximize the revenue</u>?

3. A wood screw manufacturer has determined that the total cost $C(x)$ of operating a factory is $C(x) = 0.5x^2 + 45x + 10000$, where $x$ is the number of wood screws produced in **thousands**. Determine the production level that <u>minimizes the</u> <u>average cost</u>.

   (Note: Average Cost is found by dividing Total Cost by # of items… or —— )

4. A soft drink company estimates that the cost, $C(x)$, in dollars of producing $x$ litres of soda per day is $C(x) = 0.0005x^2 + 4x + 4000$. Find the <u>marginal cost</u> when 5000L of soda is produced.

5. A local manufacturer sells cedar deck chairs. The company can sell $x$ chairs at a price of $p(x) = 100 - x$ dollars, where the cost function is, $C(x) = 30 + 19x^2$. Determine the number of chairs that need to be produced to <u>break-even.</u>